Response of plasma electrons to a spatially embedded electric field impulse

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An efficient method for solving the one-dimensional inhomogeneous Boltzmann equation has been developed. This method is used to investigate in a helium plasma the response of the electron velocity distribution function (EVDF) and of relevant macroscopic quantities to the impact of a spatially embedded disturbance in the electric field. Both elastic and conservative inelastic collisions of electrons with gas atoms are taken into account. The EVDF and macroscopic quantities show unexpectedly large spatial relaxation lengths in electron acceleration direction and large deviations from those obtained in the local field approximation.

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INTRODUCTION

The spatially resolved theoretical treatment of the electron kinetics in weakly ionized, nonequilibrium plasmas is of fundamental interest for the understanding of various phenomena occurring in inhomogeneous plasma regions as well as for the modeling of real plasma devices. Inhomogeneous plasma regions occur, e.g., (i) in front of discharge electrodes, (ii) in the interface between the plasma and its enclosing wall, and (iii) in plasmas due to the presence of grids, constrictions, or probes leading to spatial disturbances or to standing and moving striations. Concerning the kinetics in electrode regions, which are characterized by large electric fields with remarkable change in coordinate space, many studies (e.g., [1,2]) have already been performed in the past. However, with respect to the other problems (ii) and (iii) until now only some efforts [3,4] and [5–7], with more or less success, have been made to discover and to understand the electron kinetics and the global plasma behavior in spatially structured plasmas. This study concerns the electron kinetics relevant to the problems (iii). In the following an efficient method for solving the one-dimensional Boltzmann equation is briefly represented. This method is applicable to a variety of electric field courses. As a first example it is used to study the response of the electrons to field impulses embedded between homogeneous states of the electric field.

KINETIC EQUATION AND MACROSCOPIC BALANCES

The starting point is the spatially inhomogeneous stationary Boltzmann equation

$$\vec{v}\cdot\nabla_{\vec{r}}f-\frac{e_0}{m}\vec{E}\cdot\nabla_{\vec{v}}f=C^{el}(f)+\sum_k C_k^{in}(f) \qquad (1)$$

for the velocity distribution $f(\vec{v},\vec{r})$ of the electrons (charge $-e_0$, mass m), which includes the action of a space dependent electric field $\vec{E}=E(z)\vec{e}_z$ and the collision integrals of elastic (C^{el}) and various conservative inelastic (C_k^{in}) collision processes. The electron velocity distribution function (EVDF) is approximated by the first two terms of its expansion in Legendre polynomials

$$f\left(v,\frac{v_z}{v},z\right)=\tilde{f}_0(v,z)+\tilde{f}_1(v,z)\frac{v_z}{v},\quad v=|\vec{v}| \qquad (2)$$

where $\tilde{f}_0$ and $\tilde{f}_1$ denote the isotropic and anisotropic part of the EVDF. If introducing (2) into (1) and substituting the electron velocity v by the kinetic energy $U=\frac{m}{2}v^2$, the following partial differential equation system for the functions $f_j(U,z)=2\pi(2/m)^{3/2}\tilde{f}_j(v(U),z), j=0,1$ is obtained:

$$\frac{\partial}{\partial z}(Uf_1)-e_0E(z)\frac{\partial}{\partial U}(Uf_1)+\frac{\partial}{\partial U}[C(U)f_0]+\sum_k F_k(U)f_0=\sum_k F_k(U+U_k^{in})f_0(U+U_k^{in},z),$$

$$\frac{\partial}{\partial z}f_0-e_0E(z)\frac{\partial}{\partial U}f_0+H(U)f_1=0. \qquad (3)$$

The coefficients $C(U)=-6(m/M)U^2NQ^d(U)$, $F_k(U)=3UNQ_k^{in}(U)$, and $H(U)=NQ^d(U)+\sum_k NQ_k^{in}(U)$ are determined by the atomic data of the collision processes and by the density N and mass M of the gas atoms. Q^d is the transport cross section of elastic collisions and Q_k^{in} the total cross section of the kth inelastic collision process with the energy loss U_k^{in} assuming isotropic scattering in the latter.

In order to understand the macroscopic electron behavior as well as to check the accuracy of the numerical solutions of the kinetic equation system (3), the consistent macroscopic balances are of particular importance. Appropriate averaging of the first equation of (3) over the energy space yields the particle balance

$$\frac{d}{dz}j_z(z)=0 \qquad (4)$$

and the energy balance

$$\frac{d}{dz}j_u(z)=\langle U^f\rangle(z)-\langle U^{el}\rangle(z)-\sum_k\langle U_k^{in}\rangle(z)\ , \qquad (5)$$

with the current density

$$j_z = \frac{1}{3}\sqrt{2/m}\int_0^\infty U f_1(U,z)dU,$$

the energy current density

$$j_u(z) = \frac{1}{3}\sqrt{2/m}\int_0^\infty U^2 f_1(U,z)dU, \tag{6}$$

the energy gain from the electric field

$$\langle U^f\rangle(z) = -j_z e_0 E(z),$$

the energy loss by elastic collisions

$$\langle U^{el}\rangle(z) = 2\frac{m}{M}\sqrt{2/m}\int_0^\infty U^2 N Q^d(U) f_0(U,z)dU,$$

and the energy loss by inelastic collisions

$$\langle U_k^{in}\rangle(z) = U_k^{in}\sqrt{2/m}\int_0^\infty U N Q_k^{in}(U) f_0(U,z)dU.$$

System (3) can be simplified by substituting the total energy $\varepsilon = U + W(z)$ for the kinetic energy U, where the potential energy is introduced according to $W(z) = -\int_0^z E(\tilde{z})d\tilde{z}(-e_0)$. When using the definitions $\bar{f}_j(\varepsilon,z) = f_j(U(\varepsilon,z),z)$, $j=0,1$, where $U(\varepsilon,z) = \varepsilon - W(z)$, and when eliminating the anisotropic distribution $\bar{f}_1$ the parabolic partial differential equation in standard form

$$-\frac{\partial}{\partial z}\left(\frac{[\varepsilon - W(z)]}{H[\varepsilon - W(z)]}\frac{\partial}{\partial z}\bar{f}_0\right) + \frac{\partial}{\partial\varepsilon}\{C[\varepsilon - W(z)]\bar{f}_0\} + \sum_k F_k[\varepsilon - W(z)]\bar{f}_0 = \sum_k F_k[\varepsilon - W(z) + U_k^{in}]\bar{f}_0(\varepsilon + U_k^{in}, z) \tag{7}$$

for $\bar{f}_0$ is finally obtained. The additional difference terms $\bar{f}_0(\varepsilon + U_k^{in}, z)$ on the right-hand side describe the inscattering of electrons at the energy ε which have undergone inelastic collisions at the higher energy $\varepsilon + U_k^{in}$. The anisotropic distribution is determined by the solution $\bar{f}_0$ of (7) according to the relation

$$\bar{f}_1(\varepsilon,z) = -\frac{1}{H[\varepsilon - W(z)]}\frac{\partial}{\partial z}\bar{f}_0(\varepsilon,z).$$

NUMERICAL SOLUTION METHOD

The transformation from kinetic to total energy changes the solution region. Figure 1 illustrates its nonrectangular shape limited by the vertical line A and the curves B and C which depend on the course of the electric field. If choosing negative values for the electric field $E(z)$ in the entire space range, the curves B and C monotonously decrease with z and the electrons are accelerated towards higher z values. According to the parabolic nature of (7), the evolution direction of its solution is the total energy ε. Equation (7) can be solved as an initial boundary value problem. If the field disturbance region is sufficiently far from the position $z=0$, the boundary value at $z=0$ is determined by the solution of the homogeneous Boltzmann equation. On curve B the spatial derivative of $\bar{f}_0$ should be zero. This is equivalent to the condition that the anisotropic distribution vanishes at zero kinetic energy. On curve C, i.e., at a sufficiently high kinetic energy U^∞, $\bar{f}_0$ is assumed to be negligible. To solve the parabolic problem a discretization of (7) has been performed on an equidistant grid in z and a nonequidistant grid in ε using a modified Crank-Nicholson technique. The resultant discrete equation system is resolved from higher to lower total energies. Choosing this direction, the inscattering terms $\bar{f}_0(\varepsilon + U_k^{in}, z)$ of the isotropic distribution at a certain ε position can be directly obtained from the distribution function already calculated at higher total energies.

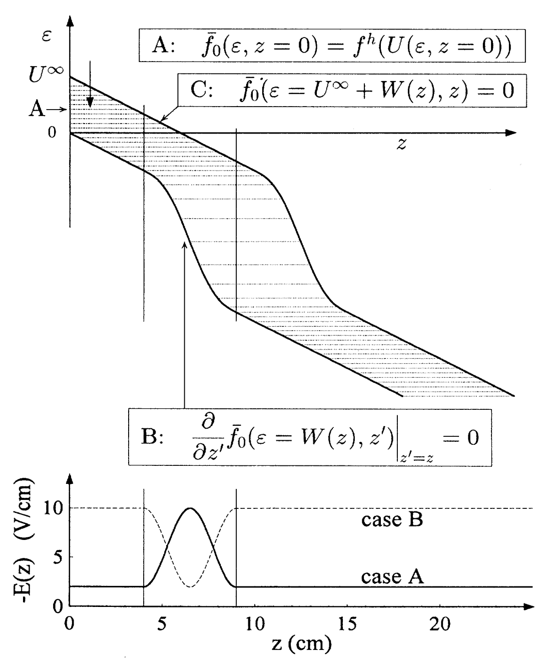

FIG. 1. Solution region in transformed variables, initial boundary values, and the two considered electric field courses A and B.

RESULTS

The solution method outlined above is used to investigate in a helium plasma the response of the electrons to the two different electric field pulses A and B displayed in Fig. 1. The normalization of system (3) by the gas pressure p at 0 °C is possible and would lead to the normalized quantities E/p and zp. However, in the following, all results are represented for a gas pressure of 1 torr using the natural quantities E and z. The data of the transport cross section Q^d have been taken from [8,9]. Four inelastic collision processes are considered using the cross sections from [10,11] for excitation and the ionization cross sections from [12]. The latter process has

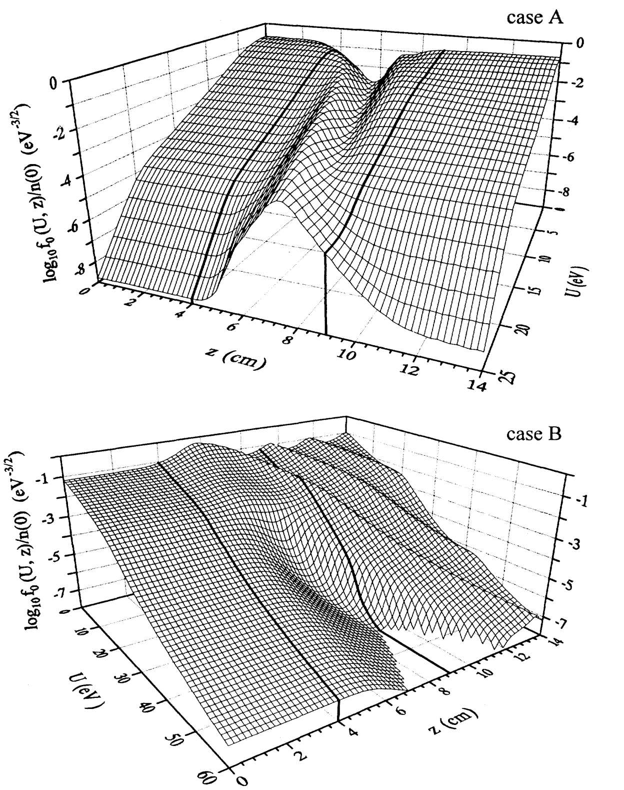

FIG. 2. The isotropic distribution normalized on the electron density $n(z=0)$ as a function of the kinetic energy and space coordinate.

been dealt with as an excitation process.

The spatial evolution of the isotropic distribution $f_0(U,z)$ in the upstream and downstream area close to the field inhomogeneity region is shown in Fig. 2 for cases A and B starting from the homogeneous state at $z=0$. The spatial borders of the field disturbance are marked by thick lines. Particularly in case B the isotropic distribution is in the right-hand part of Fig. 2, still far from its establishment in the homogeneous state. A quite different response of the isotropic distribution can be observed in both cases. Contrary to the aperiodic establishment with increasing z in case A, a distinct periodic structure is excited in case B. This periodic behavior is mainly caused by the interplay of field acceleration and backscattering in inelastic collisions, which is more pronounced in case B. The period length is approximately given by $\lambda \approx U^{th}/(e_0\tilde{E})$, where U^{th} corresponds to the lowest energy losses U_k^{in} of the inelastic collision processes and $\tilde{E}$ is the undisturbed electric field strength.

Figure 3 compares the isotropic distribution determined from the inhomogeneous Boltzmann equation with that calculated by the local field approximation. The latter is obtained by solving the homogeneous Boltzmann

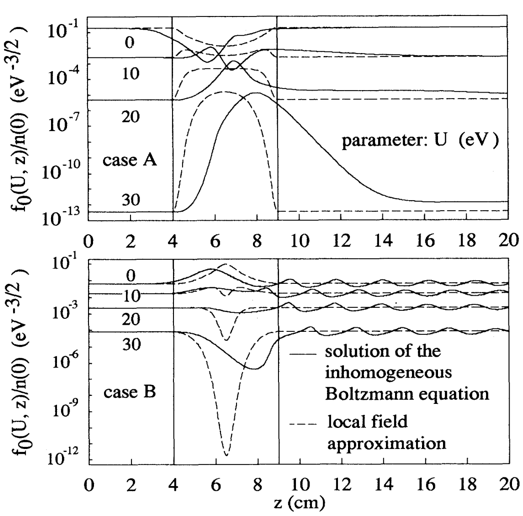

FIG. 3. The solution of the inhomogeneous Boltzmann equation in comparison with the local field approximation as a function of the space coordinate for different kinetic energies.

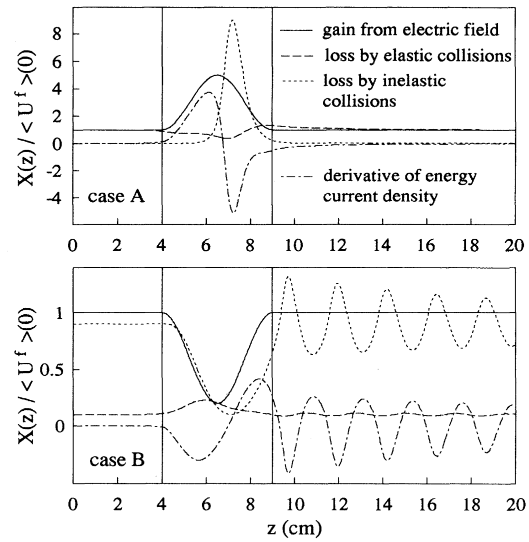

FIG. 4. Normalized energy balance terms $X(z)/\langle U^f\rangle(0)$, where $X = dj_u/dz$, $\langle U^f\rangle$, $\langle U^{el}\rangle$ and $\sum_k \langle U_k^{in}\rangle$.

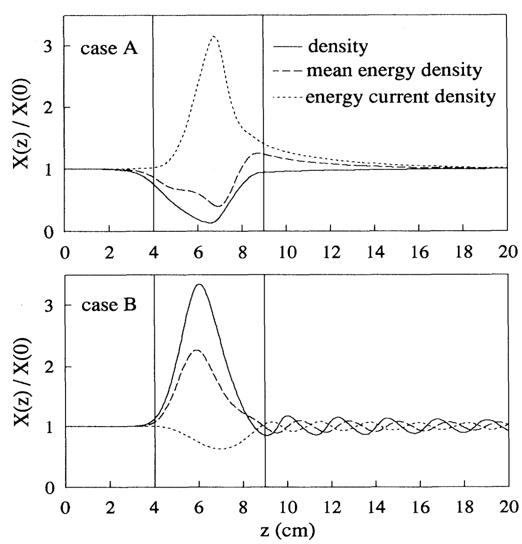

FIG. 5. Normalized macroscopic quantities $X(z)/X(0)$, where $X = n$, $\langle U\rangle$, and j_u.

equation for those electric field values $E(z)$ which are assumed in the course of the field through the field disturbance region and by normalizing the distribution function on the constant current density according to (4). Large deviations of the isotropic distribution in local field approximation from the strict solution become obvious inside the field inhomogeneity region and in the further establishment region at higher z values. Notice that a slight spatial dependence of the strict solution occurs even in the upstream direction at lower energies. In both cases the acceleration of electron groups can be observed. Whereas in case B further electron groups are generated by repeated backscattering and acceleration, in case A the more pronounced impact of elastic collisions rapidly suppresses the generation of further distinct groups.

Figure 4 represents the various terms of the energy balance (5) for both cases. Because of the constant current density (4) the course of the energy gain directly reflects that of the electric field. In case A the energy loss outside the field inhomogeneity is mainly caused by elastic collisions whereas in case B the loss by inelastic collisions dominates in these regions. Mainly this change of the dominant loss channel leads to a quite different energetic relaxation behavior. While in case A the homogeneous state is nearly reached at 20 cm, the distinct periodic structure in case B is still present at this position and the final establishment requires about 50 cm. The large contribution of the derivative term to the energy balance (5) underlines once more the strong violation of the local field approximation under these conditions.

Figure 5 shows the space dependence of the density $n(z) = \int_0^\infty U^{1/2} f_0(U,z)dU$, the mean energy density $\langle U\rangle(z) = \int_0^\infty U^{3/2} f_0(U,z)dU$, and the energy current density (6) of electrons. A relaxation behavior similar to that found for the isotropic distribution and for the various contributions to the energy balance can be observed from the spatial course of these macroscopic quantities.

CONCLUSION

A powerful method for solving the inhomogeneous Boltzmann equation has been developed. This method is used to study the response of electron kinetic quantities in a helium plasma to spatially embedded field impulses. Long living periodic structures are excited in the case of dominant energy loss by inelastic collisions in the downstream region. Unexpectedly large deviations from the local field approximation have been found in the spatial relaxation process.

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